## Computer Graphics

## 10 - Kinematics \& Animation

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Spring 2022

## Topics Covered

- Forward Kinematics
- Introduction to Character Animation
- Motion Capture Data


## Forward Kinematics

## Kinematics

- Kinematics is
- Study of motion of objects (or groups of objects), without considering mass or forces
- In computer graphics, it's about how to move skeletons
- Forward kinematics
- Inverse kinematics

- By contrast, Dynamics (or Kinetics) is
- Study of the relationship between motion and its causes, specifically, forces and mass


## Kinematics



$$
(\mathbf{p}, \mathbf{q})=\mathrm{F}\left(\theta_{i}\right)
$$

Forward Kinematics
: Given joint angles, compute the position \& orientation of end-effector


$$
\theta_{i}=\mathrm{F}^{-1}(\mathbf{p}, \mathbf{q})
$$

Inverse Kinematics
: Given the position \& orientation of end-effector, compute joint angles

## Articulated Body

- A common type of hierarchical model used in CG is an articulated body
- that has objects that are connected end to end to form multibody jointed chains.
- a.k.a. kinematic chain, linkage (robotics)

- Terminologies
- Joint - a connection between two objects which allows some motion
- Link - a rigid object between joints
- End effector - a free end of a kinematic chain



## [Practice] FK / IK Online Demo


http://robot.glumb.de/

- Forward kinematics : Open "angles" menu and change values
- Inverse kinematics : Move the end-effector position by mouse dragging


## Forward Kinematics: A Simple Example

- A simple robot arm in 2-dimensional space
- 2 revolute joints
- Joint angles are known
- Compute the position of the end-effector


$$
\begin{aligned}
& x_{e}=l_{1} \cos \theta_{1}+l_{2} \cos \left(\theta_{1}+\theta_{2}\right) \\
& y_{e}=l_{1} \sin \theta_{1}+l_{2} \sin \left(\theta_{1}+\theta_{2}\right)
\end{aligned}
$$

## A Chain of Transformations



## Thinking of Transformations

- In a view of body-attached coordinate system (=local coordinate system of the end-effector body)

$T=\left(\operatorname{rot} \theta_{1}\right)\left(\right.$ transl $\left._{1}\right)\left(\operatorname{rot}_{2}\right)\left(\right.$ transl $\left._{2}\right)$
$=\left(\begin{array}{ccc}\cos \theta_{1} & -\sin \theta_{1} & 0 \\ \sin \theta_{1} & \cos \theta_{1} & 0 \\ 0 & 0 & 1\end{array}\right)\left(\begin{array}{ccc}1 & 0 & l_{1} \\ 0 & 1 & 0 \\ 0 & 0 & 1\end{array}\right)\left(\begin{array}{ccc}\cos \theta_{2} & -\sin \theta_{2} & 0 \\ \sin \theta_{2} & \cos \theta_{2} & 0 \\ 0 & 0 & 1\end{array}\right)\left(\begin{array}{lll}1 & 0 & l_{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1\end{array}\right)$


## Thinking of Transformations

- In a view of body-attached coordinate system

$T=\left(\operatorname{rot} \theta_{1}\right)\left(t r a n s l_{1}\right)\left(\operatorname{rot} \theta_{2}\right)\left(\right.$ transl $\left._{2}\right)$
$=\left(\begin{array}{ccc}\cos \theta_{1} & -\sin \theta_{1} & 0 \\ \sin \theta_{1} & \cos \theta_{1} & 0 \\ 0 & 0 & 1\end{array}\right)\left(\begin{array}{lll}1 & 0 & l_{1} \\ 0 & 1 & 0 \\ 0 & 0 & 1\end{array}\right)\left(\begin{array}{ccc}\cos \theta_{2} & -\sin \theta_{2} & 0 \\ \sin \theta_{2} & \cos \theta_{2} & 0 \\ 0 & 0 & 1\end{array}\right)\left(\begin{array}{lll}1 & 0 & l_{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1\end{array}\right)$


## Thinking of Transformations

- In a view of body-attached coordinate system

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## Thinking of Transformations

- In a view of body-attached coordinate system

$T=\left(\operatorname{rot} \theta_{1}\right)\left(\right.$ trans $\left._{1}\right)\left(\right.$ rot $\left.\theta_{2}\right)\left(t r a n s l_{2}\right)$
$=\left(\begin{array}{ccc}\cos \theta_{1} & -\sin \theta_{1} & 0 \\ \sin \theta_{1} & \cos \theta_{1} & 0 \\ 0 & 0 & 1\end{array}\right)\left(\begin{array}{ccc}1 & 0 & l_{1} \\ 0 & 1 & 0 \\ 0 & 0 & 1\end{array}\right)\left(\begin{array}{ccc}\cos \theta_{2} & -\sin \theta_{2} & 0 \\ \sin \theta_{2} & \cos \theta_{2} & 0 \\ 0 & 0 & 1\end{array}\right)\left(\begin{array}{lll}1 & 0 & l_{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1\end{array}\right)$


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## Thinking of Transformations

- In a view of global coordinate system


$$
\begin{aligned}
T & =\left(\text { rot } \theta_{1}\right)\left(\text { transl }_{1}\right)\left(\text { rot } \theta_{2}\right)\left(\text { transl }_{2}\right) \\
& =\left(\begin{array}{ccc}
\cos \theta_{1} & -\sin \theta_{1} & 0 \\
\sin \theta_{1} & \cos \theta_{1} & 0 \\
0 & 0 & 1
\end{array}\right)\left(\begin{array}{ccc}
1 & 0 & l_{1} \\
0 & 1 & 0 \\
0 & 0 & 1
\end{array}\right)\left(\begin{array}{ccc}
\cos \theta_{2} & -\sin \theta_{2} & 0 \\
\sin \theta_{2} & \cos \theta_{2} & 0 \\
0 & 0 & 1
\end{array}\right)\left(\begin{array}{lll}
1 & 0 & l_{2} \\
0 & 1 & 0 \\
0 & 0 & 1
\end{array}\right)
\end{aligned}
$$

## Thinking of Transformations

- In a view of global coordinate system

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- In a view of global coordinate system


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\end{array}\right)\left(\begin{array}{ccc}
1 & 0 & l_{1} \\
0 & 1 & 0 \\
0 & 0 & 1
\end{array}\right)\left(\begin{array}{ccc}
\cos \theta_{2} & -\sin \theta_{2} & 0 \\
\sin \theta_{2} & \cos \theta_{2} & 0 \\
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## Thinking of Transformations

- In a view of global coordinate system

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## Thinking of Transformations

- In a view of global coordinate system

$T=\left(r o t \theta_{1}\right)\left(\right.$ trans $\left._{1}\right)\left(\operatorname{rot} \theta_{2}\right)\left(\right.$ trans $\left._{2}\right)$
$=\left(\begin{array}{ccc}\cos \theta_{1} & -\sin \theta_{1} & 0 \\ \sin \theta_{1} & \cos \theta_{1} & 0 \\ 0 & 0 & 1\end{array}\right)\left(\begin{array}{ccc}1 & 0 & l_{1} \\ 0 & 1 & 0 \\ 0 & 0 & 1\end{array}\right)\left(\begin{array}{ccc}\cos \theta_{2} & -\sin \theta_{2} & 0 \\ \sin \theta_{2} & \cos \theta_{2} & 0 \\ 0 & 0 & 1\end{array}\right)\left(\begin{array}{ccc}1 & 0 & l_{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1\end{array}\right)$


## Quiz \#1

- Go to https://www.slido.com/
- Join \#cg-ys
- Click "Polls"
- Submit your answer in the following format:
- Student ID: Your answer
- e.g. 2017123456: 4)
- Note that you must submit all quiz answers in the above format to be checked for "attendance".


## Forward Kinematics Map

- A forward kinematics map is a mapping from joint angles to end effector position \& orientation.
- Usually, a forward kinematics map T is an alternating multiple of ...
- Joint transformations that represents joint movement (time-varying)
- Usually rotations
- Link transformations that defines a frame relative to its parent (static)
- Usually translations (joint offset from its parent joint)



## Forward Kinematics Map

current frame
$T=\mathrm{I}$


## Forward Kinematics Map

$$
T=J_{0}
$$



## Forward Kinematics Map

$$
T=J_{0} L_{1}
$$



## Forward Kinematics Map

$$
T=J_{0} L_{1} J_{1}
$$



## Forward Kinematics Map

$$
T=J_{0} L_{1} J_{1} L_{2}
$$



## Forward Kinematics Map

$$
T=J_{0} L_{1} J_{1} L_{2} J_{2}
$$



## Forward Kinematics Map

$$
T=J_{0} L_{1} J_{1} L_{2} J_{2} L_{3}
$$



## Forward Kinematics Map

$$
T=J_{0} L_{1} J_{1} L_{2} J_{2} L_{3} J_{3}
$$



## Forward Kinematics Map

$$
p_{o}{ }^{\{g\}}=J_{0} L_{1} J_{1} L_{2} J_{2} L_{3} J_{3} \frac{p_{o}^{\{3\}}}{(0,0,0)}
$$



## Forward Kinematics Map

$$
p_{a}{ }^{\{g\}}=J_{0} L_{l} J_{1} L_{2} J_{2} L_{3} J_{3} p_{a}^{\{3\}}
$$



## Quiz \#2

- Go to https://www.slido.com/
- Join \#cg-ys
- Click "Polls"
- Submit your answer in the following format:
- Student ID: Your answer
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# Introduction to Character Animation 

## Traditional Hand-drawn Cel Animation

- Senior artist draws keyframes
- Assistant draws inbetweens
- Tedious / labor intensive (opportunity for technology!)



Animation by Milt Kahl (Walt Disney Studios)


Animation by Mark Henn (Walt Disney Studios)


Animation by Marc Davis (Walt Disney Studios)


## Computer Animation

- Computers are now widely replacing laborintensive animation processes.
- More controllable than drawing images by hands or constructing miniatures.


## Character Animation Approaches

- Keyframe Animation
- Motion Capture
- Data-Driven Animation
- Deep Motion Synthesis
- Physics-Based Animation
- Learning Deep Control Policies


## Keyframe Animation

- Idea:
- Animators specifies important events at key frames.
- Computer fills inbetween frames using interpolation.
- Events can be
- Positions \& orientation of objects
- Light intensities
- Camera parameters




## Keyframe Animation

- One of the earliest methods used to produce computer animation.
- Difficult to create "realistic" and "physically plausible" motions.
- The quality of the output largely depends on the skill of the individual artist.
- Still used a lot.


## Motion Capture

- Idea: Use "real" human motion to create realistic animation.
- Motion capture system "captures" movement of people or objects by measuring
- position of each marker on the skin
- position and orientation of each body part (or joint)


## Motion Capture


https://youtu.be/YzS73UCOk20

## Motion Capture

- Currently, widely used in movies \& games
- by major companies
- Very expansive
- Expensive devices
- High operating cost
- Limitation: Motion captured data is very realistic only in the same virtual environment as capture environment.


## Data-Driven Animation

- Idea: Make the most of the motion capture data by
- reusing mocap data with various motion editing techniques
- or learning a machine learning model with mocap data
- to generate new motions.
- Recently, deep learning techniques are used a lot.
$-\rightarrow$ Deep motion synthesis


## Deep Motion Synthesis

- Learning a model generating new motions using
- Discriminative models with FFNN, CNN, or RNN structures
- Generative models with GAN, VAE, or Flow
- Manifold learning with Autoencoders
- Reinforcement learning to learn a policy
- ...


## Example

## Physics-Based Animation

- Idea: Use physics simulation to generate motion
- Because physical reality plays a key role in creating high-quality motion.
- Physic simulation generates a motion that is always physically plausible.
- This requires a "controller".
- Determines joint torques at each timestep to perform desired action while maintaining balance.
- This problem is similar to that of robotics.
- Recently, deep reinforcement learning (DRL) is used a lot to learn deep control policies.


## Example



Kwon, Taesoo, Yoonsang Lee, and Michiel Van De Panne. "Fast and Flexible Multilegged Locomotion Using Learned Centroidal Dynamics." ACM Transactions on Graphics 39, no. 4 (SIGGRAPH 2020)

Motion Capture Data

## Motion Capture Data

- Motion capture data includes two parts:
- "Skeleton": static data
- joint hierarchy
- joint offset from its parent joint - link transformation Ls
- "Motion": time-varying data
- internal joint orientation (w.r.t. default frame of each joint - the frame after applying link transformation for that joint)
- position and orientation of skeletal root (w.r.t. global frame)
- $\quad \rightarrow$ joint transformations Js
- Posture (pose): "motion" at a single frame
- T pose: a pose where all joint orientations are "zero" (identity matrix)



## Motion Capture Data

- Generally, in motion capture data,
- Internal joint has 3 DOFs
- rotation only
- Root joint has 6 DOFs
- rotation and translation



## BVH File Format

- BVH (BioVision Hierarchical data)
- Developed by Biovision, a motion capture company
- Consists of two parts:
- Hierarchy section
- Describes the "Skeleton": static data
- Motion section
- Describes the "Motion": time-varying data
- Text file format (human-readable)


## Hierarchy Section

- The hierarchy is a joint tree.
- Each joint has
- Offset from its parent joint link transformation L
- Channel list, which defines
 the type of its joint transformation J

```
|IFMRCT

\section*{Biovision BVH}

\section*{\(\begin{array}{llll}\text { OFFSE } & 3.430000 & 0.000000 & 0.000000\end{array}\)}

CHisels 3 Zrotation Irotation Irotation

\section*{JOIT Leftriee}
\{
```

                                    OFFCT 0.00000 
    ```
                CIHELS 3 Zrotation Irvation Yrotation
                IOIT Leftiakle
                \{
                    OFFFET \(\quad 0.000000-17.95001 \quad 0.000000\)
                Cilime. 3 Irotation Iretation Yrotation
                End Site
            \(\{\)
                    OFFST \(\quad 0.00000\)-3.119099 0.00000
                    \}
                \}
\}

Hierarchy section
> "HIERARCHY"
- "ROOT"
- followed by the name of the root
- "\{" and"\}"pair
_ "OFFSET"
» \(X, Y\) and \(Z\) offset of the segment from its parent
_ "CHANNELS"
» the number of channels
" the type of each channel
- "JOINT"
- identical to the root definition except for the number of channels
_ "OFFSET", "CHANNELS"
- "End Site"
- indicates that the current segment is an end effector (no children)
- "OFFSET"
- 6 channels for the root (Tx Ty Tz Rz Rx Ry)
- 3 channels for every other object ( \(R z R x R y\) )

\section*{HIERARCHY} ROOT Hips
\{
OFFSET 0.00 .00 .0
CHANNELS 6 XPOSITION YPOSITION ZPOSITION ZROTATION XROTATION YROTA
JOINT chest
\{
```

OFFSET 0.096536 3.475309 -0.289609
CHANNELS 3 Xrotation Zrotation Yrotation
TOINT neck
{
OFFSET -0.096536 13.901242 -2.027265
CHANNELS 3 Xrotation Zrotation Yrotation
IOINT head
{
OFFSET -0.166775 1.448045 0.482682
CHANNELS }3\mathrm{ Xrotation Zrotation Yrotation
TOINT leftEye

```

\section*{HIERARCHY}

\section*{ROOT Hips}
\{
OFFSET 0.00 .00 .0 JO channels
CHANNELS 6 XPOSITION YPOSITION ZPOSITION ZROTATION XROTATION YROTA
JoINT chest
\{
```

OFFSET 0.0196536 3.475309 -0.289669 L1
CHANNELS 3 Xrotation Zrotation Yrotation J1 channels
IOINT neck
{
OFFSET -0.096536 13.901242 -2.027265 L2
CHANNELS 3 Xrotation Zrotation Yrotation J2 channels
IOINT head
{
OFFSET -0.166775 1.448045 0.482682 L3
CHANNELS 3 Xrotation Zrotation Yrotation
TOINT leftEye
J3 channels

```

\section*{HIERARCHY}

\section*{ROOT Hips Root Hips Joint}
\{
OFFSET 0.00 .00 .0 Root offset is generally zero (or ignored even if it's not zero) CHANNELS 6 XPOSITION YPOSITION ZPOSITION ZROTATION XROTATION YROTA JOINT chest Chest Joint
\{
```

OFFSET 0.096536 3.475309 -0.289609

```
CHANNELS 3 Xrotation Zrotation Yrotation
ZOINT neck Neck Joint
\{


Neck's offset from chest

OFFSET - 0.166775 . 4486450.482682 CHANNELS 3 Xrotation Zrotation Yrotation TOINT leftEye

Channel list:
Transformation from chest frame to neck frame

■ Motion Section
> "MOTION"
- followed by a line indicating the number of frames
- "Frames:"
- the number of frames
- "Frame Time:"
- the sampling rate of the data
- Ex) \(0.033333 \rightarrow 30\) frames a second
- The rest of the file contains the actual motion data
- The numbers appear in the order of the channel specifications as the skeleton hierarchy was parsed
- Each line has motion data for a single frame
- Each number in a line is a value for a single channel
- The unit of rotation channel values is degree
```

HIERARCHY
ROOT Hips
{
OFFSET 0.0 0.0 0.0
CHANNELS 6 XPOSITION YPOSITION ZPOSITION ZROTATION XROTATION YROTATION

```
        JOINT chest Column 1 Column 2 Column 3 Column 4 Column 5 Column 6
        \{
        OFFSET 0.696536 3.475369-0.2896699
        CHANNELS 3 Xrotation Zrotation Yrotation
                JOINT neck Column 7 Column 8 Column 9
                โ
                    OFFSET - \(0.09653613 .901242-2.027265\)
                CHANNELS 3 Xrotation Zrotation Yrotation
                JOINT head Column 10 Column 11 Column 12
\{
                OFFSET - 1.1667751 .4486450 .482682
                CHANNELS 3 Xrotation Zrotation Yrotation
                        Column 13 Column 14 Column 15
MOTION
Frames: 199
Frame Time: 0.033333
\(1.957690 .9897694793210 .039193-4.11275998891-0.490682977769-91.35199746950 .45458697547\)
\(1.957690 .9897694793210 .0392908-4.11760985011-0.48626597981-91.37349890510 .513819016282\)
\(1.957690 .9897694793210 .039424-4.12004011679-0.488125979059-91.3870021890 .592700017233\)
\(1.957710 .9897694793210 .0395518-4.0961698863-0.500940000911-91.38409935860 .61126399115\)...
\(1.957790 .9897594793210 .0396999-4.05799980101-0.510696019006-91.38399690580 .58299101005\)
\(1.95790 .9897194793210 .0398625-4.0423300664-0.503295989288-91.38420181150 .57718001317\)...

\section*{Biovision BVH}

\section*{■ Interpreting the data}
\(>\) To calculate the position of a segment
- Translation information
- For any joint segment
" the translation information will simply be the offset as defined in the hierarchy section
- For the root object
» The translation data will be the sum of the offset data and the translation data from the motion section
- Rotation information
- comes from the motion section
- The "CHANNELS" order is important: If the order is "ZROTATION XROTATION YROTATION"
- Apply transformation in order of rotation about \(z\), rotation about \(x\), rotation about \(y\) w.r.t. local frame
- \(\rightarrow\) ZXY Euler angles
- Do not assume ZXY Euler angles. Other sequences may also be used.

\section*{[Practice] BVH Online Demo}
```

three.js - BVH Loader - animation from http://mocap.cs.cmu.edu/

```

http://motion.hahasoha.net/
- Select other motions from the list.
- Download corresponding BVH files and open them in a text editor.

\section*{Quiz \#3}
- Go to https://www.slido.com/
- Join \#cg-ys
- Click "Polls"
- Submit your answer in the following format:
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\section*{Next Time}
- Lab for this lecture (next Monday):
- Lab assignment 10
- Next lecture:
- 11 - Curve
- Class Assignment \#3
- Due: 23:59, Jun 7, 2021
- Acknowledgement: Some materials come from the lecture slides of
- Prof. Kayvon Fatahalian and Prof. Keenan Crane, CMU, http://15462.courses.cs.cmu.edu/fall2015/
- Prof. Jinxiang Chai, Texas A\&M Univ., http://faculty.cs.tamu.edu/jchai/csce441 2016spring/lectures.html
- Prof. Jehee Lee, SNU, http://mrl.snu.ac.kr/courses/CourseGraphics/index 2017spring.html
- Prof. Taesoo Kwon, Hanyang Univ., http://calab.hanyang.ac.kr/cgi-bin/cg.cgi```

